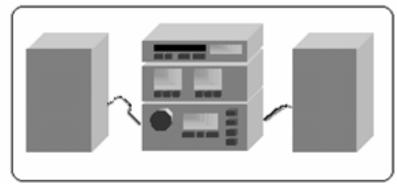


INTRODUCTION TO AC FILTERS AND RESONANCE



OBJECTIVES

- To understand the design of capacitive and inductive filters
- To understand resonance in circuits driven by AC signals

OVERVIEW

In a previous lab, you explored the relationship between impedance (the AC equivalent of resistance) and frequency for a resistor, capacitor, and inductor. These relationships are very important to people designing electronic equipment, particularly audio equipment. You can predict many of the basic characteristics of simple audio circuits based on what you have learned in previous labs.

Recall that if there is a current of the form

$$I(t) = I_{\max} \sin(\omega t)$$

flowing through a circuit containing resistors, capacitors and/or inductors, then the voltage across the circuit will be of the form

$$V(t) = I_{\max} Z \sin(\omega t + \varphi)$$

Z is called the *impedance* and φ is called the *phase shift*. The maximum voltage will be given by

$$V_{\rm max} = I_{\rm max} Z$$
 .

When φ is zero, the voltage and current will be in phase. When φ is less than zero, the voltage will reach its peak before the current and we say that the current *lags* the voltage. When φ is greater than zero, the voltage will reach its peak after the current and we say that the current *leads* the voltage.

For a series combination of a resistor, a capacitor and an inductor,

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

and

$$\tan(\varphi) = \frac{X_L - X_C}{R}$$

where

 $X_c \equiv \frac{1}{\omega C}$

and

$$X_L \equiv \omega L$$
.

 X_C is called the *capacitive reactance* and X_L is called the *inductive reactance*. If there is only a capacitor or only an inductor, the impedance is simply the corresponding reactance.

In this lab you will continue your investigation of the behavior of resistors, capacitors and inductors in the presence of AC signals. In Investigation 1, you will see how capacitors and inductors can act as "filters". More precisely, you will see how these elements can be used to suppress the voltage of certain frequency ranges of AC signals, while leaving other signals relatively unchanged.

In Investigation 2, you will explore the relationship between peak current and peak voltage for a series circuit composed of a resistor, inductor, and capacitor. You will also explore the phase difference between the current and the voltage. This circuit is an example of a "resonant circuit." The phenomenon of resonance is a central concept underlying the tuning of a radio or television to a particular frequency.

IMPORTANT NOTE: In the experiments today, we want to compare the current through the circuit with the voltage across it. Normally we would use a current probe, but the probe's 1 Ohm internal resistance is not negligible with respect to the circuit resistances.

All of the circuits today are *series circuits* and so the same current will flow through each element. Hence, we can (and will) simply measure the voltage drop across a resistor in the circuit and calculate the current from Ohm's law.

INVESTIGATION 1: INTRODUCTION TO AC FILTERS

The purpose of this lab is for you to create circuits that filter out AC signals with frequencies outside the range of interest. In the context of these activities, a filter is a circuit that attenuates the voltage of some range of signal frequencies, while leaving other frequency ranges relatively unaffected.

You will need the following materials:

- Voltage probe
- Multimeter
- RLC Circuit Board
- Alligator clip leads

Activity 1-1: Capacitors as Filters

In this activity, you will investigate how a circuit containing a resistor, capacitor, and signal generator responds to signals at various frequencies.

Consider the circuit in Figure 1-1 with a resistor, capacitor, signal generator and voltage probe.

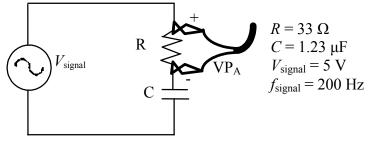
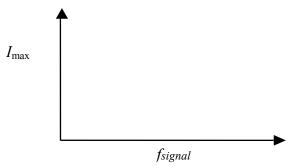


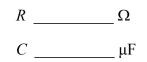
Figure 1-1: Capacitive filter circuit

Prediction 1-1: On the axes that follow, use dashed lines to sketch your *qualitative* prediction for the peak current through the circuit, I_{max} , as the frequency of the signal from the signal generator is increased from zero. [Remember that $\omega = 2\pi f$.]



Test your predictions.

- 1. Open the experiment file **AC Filter**.
- 2. We will use the internal signal generator of the computer interface. Note that the signal generator parameters will appear on the computer screen.
- 3. The signal generator should already be set to a frequency of 200 Hz and amplitude of 5 V (+5 V maximum and -5 V minimum).
- 4. Before setting up the circuit, use the multimeter to measure the value of the resistor, R, and the capacitor, C.



- 5. Connect the resistor, capacitor, signal generator and probe as shown in Figure 1-1.
- 6. Press **Start** to turn on the scope display.
- 7. You should see two displays on the scope display. One will be the voltage produced by the signal generator. This is the input (source) voltage for the circuit. It should be 5 V at its peak. The other voltage, sensed by VP_A, will be the voltage across the resistor R and is proportional to the current through the circuit.

Remember, we are *explicitly* using the voltage across **R** to measure the current through the circuit.

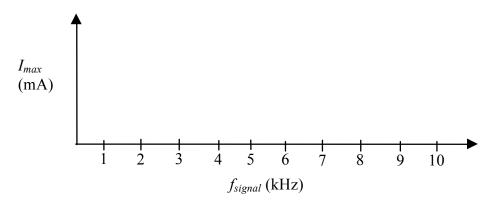
- 8. You may need to adjust the time and voltage scales on the scope display so that both the waveforms are visible. You may also need to adjust the trigger level on the left part of the screen to see the waveforms. Play with the trigger level a bit to see how it operates.
- 9. Use the **Smart Tool** to determine the peak (maximum) voltage, V_{max} , across the resistor (not the signal voltage, which should remain at 5 V), write it in **Table 1-1** with f = 200 Hz. Then calculate the maximum current from the maximum voltage using the value of the resistor you measured in step 4.
- 10. Increase the frequency of the signal generator to 1,200 Hz. Be sure that the peak signal amplitude is still 5 V. Repeat step 9.

11. Repeat step 9 for 2,200 Hz, 4,200 Hz and 8,200 Hz.

f_{signal} (Hz)	$V_{\rm max}$ (V)	$I_{\max}(\mathbf{A})$

Table 1-1

12. Sketch the data from **Table 1-1** on the axes below. Mark scales on the vertical axes.



Question 1-1: If you could continue taking data up to very high frequencies, what would happen to the peak current, I_{max} through the circuit?

Question 1-2: At very high frequencies, does the capacitor act more like an open circuit (a break in the circuit's wiring) or more like a short circuit (a connection with very little resistance)? Justify your answer.

13. Now note the phase difference (in the next question) between the peaks of the signal generator voltage and the voltage across the resistor (~circuit current) at the frequency 8,200 Hz that you should still have (note that they should be close to being in phase). Then go back to a frequency of 200 Hz and observe the phase difference.

Question 1-3: What phase difference do you observe between the peaks of the signal voltage and circuit current for low and high frequency?

Question 1-4: What would the current be through the circuit if we applied only a DC voltage? Explain.

Question 1-5: At very low frequencies, does the capacitor act more like an open circuit (a break in the circuit's wiring) or more like a short circuit (a connection with very little resistance)? Justify your answer.

Comment: In the circuit in Figure 1-1, since the peak signal voltage from the signal generator remains unchanged, the peak current in the circuit must increase as the total impedance decreases. Therefore, the peak voltage across the resistor increases as the frequency of the signal increases. This type of circuit is an example of a "high-pass" circuit or filter.

Activity 1-2: Inductors as Filters

This activity is very similar to the previous one except that you will replace the capacitor with an inductor and determine the filtering properties of this new circuit.

Consider the circuit containing a resistor, inductor, signal generator and probes shown in Figure 1-2 below.

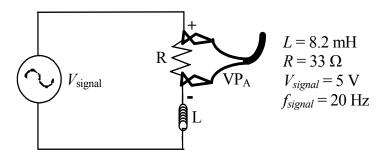
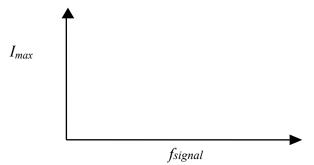


Figure 1-2: Inductive Filter Circuit

Prediction 1-2: On the axes that follow, use dashed lines to sketch your *qualitative* prediction for the peak current through the circuit, I_{max} , as the frequency of the signal from the signal generator is increased from zero.



Test your predictions.

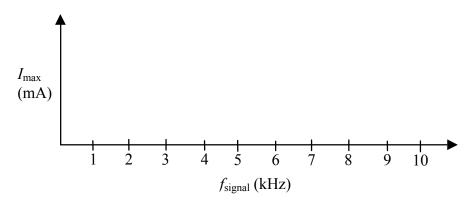
- 1. You can continue to use the experiment file AC Filter.
- 2. Set the signal generator to a frequency of 20 Hz and amplitude of 5 V.
- 3. Before setting up the circuit, use the multimeter to measure the inductance L and resistance R_L of the inductor

L _____ mH R_L _____ Ω

- 4. Connect the resistor, inductor, signal generator and probe as shown in Figure 1-2. Simply replace the capacitor in the previous setup with the inductor.
- 5. Press **Start** to turn on the scope display.
- 6. Adjust the time and voltage scales on the scope so that both waveforms are visible. Remember the trigger level.
- 7. Use the **Smart Tool** to determine the peak voltage and peak current, and enter in Table 1-2. Then calculate the maximum current from the maximum voltage using the value of the resistor.

Table 1-2			
f _{signal} (Hz)	$V_{\max}(V)$	I _{max} (mA)	

- 8. Increase the frequency of the signal generator to 200 Hz. Make sure that the amplitude is still 5 V.
- 9. Repeat step 6 with 1,200 Hz, 2,200 Hz, 4,200 Hz and 8,200 Hz.
- 10. Sketch the data from **Table 1-2** on the axes below.



Question 1-6: If you could continue taking data up to very high frequencies, what would happen to the peak current, *I_{max}*, through the resistor?

Question 1-7: At very high frequencies, does the inductor act more like an open circuit (a break in the circuit's wiring) or more like a short circuit (a connection with very little resistance)? Justify your answer.

11. Now note the phase difference between the peaks of the signal voltage and the voltage across the resistor (~circuit current) at the frequency 8,200 Hz that you should still have and then go back to a frequency of 20 Hz and observe the phase difference.

Question 1-8: What phase difference do you note between the peaks of the signal voltage and circuit current for low and high frequency? Note: We did this in last week's experiment as well.

Question 1-9: What would the current through the circuit be if we applied only a DC voltage?

Question 1-10: At very low frequencies, does the inductor act more like an open circuit (a break in the circuit's wiring) or more like a short circuit (a connection with very little resistance)? Justify your answer.

Comment: In the circuit in Figure 1-2, since the peak voltage from the signal generator remains unchanged, the peak current in the circuit must decrease as the total impedance increases. Therefore, the peak voltage across the resistor decreases as the frequency of the signal increases. This type of circuit is an example of a "low-pass" circuit or filter.

INVESTIGATION 2: THE SERIES RLC RESONANT (TUNER) CIRCUIT

In this investigation, you will use your knowledge of the behavior of resistors, capacitors and inductors in circuits driven by various AC signal frequencies to predict and then observe the behavior of a circuit with a resistor, capacitor, and inductor connected in series.

The RLC series circuit you will study in this investigation exhibits a "resonance" behavior that is useful for many familiar applications. One of the most familiar uses of such a circuit is as a tuner in a radio receiver.

You will need the following materials:

- voltage probe
- RLC Circuit Board

Consider the series RLC circuit shown in Figure 2-1 (below).

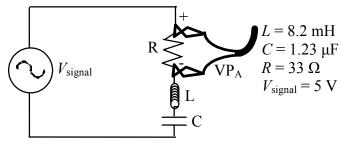


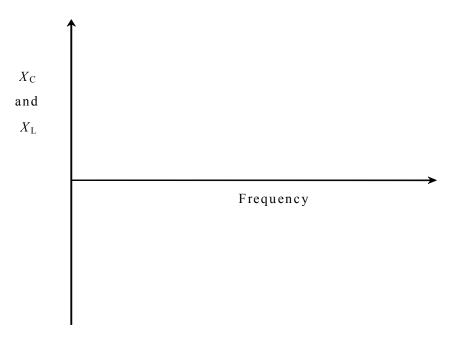
Figure 2-1: RCL Series Circuit

Prediction 2-1: At very low signal frequencies (near 0 Hz), will the maximum values of *I* through the resistor and *V* across the resistor be relatively large, intermediate or small? Explain your reasoning.

Prediction 2-2: At very high signal frequencies (well above 3,000 Hz), will the maximum values of *I* and *V* be relatively large, intermediate or small? Explain your reasoning.

Prediction 2-3: Based on your Predictions 2-1 and 2-2, is there some intermediate frequency where *I* and *V* will reach maximum or minimum values? Do you think they will be maximum or minimum?

1. On the axes below, draw qualitative graphs of $X_{\rm C}$ vs. frequency and $X_{\rm L}$ vs. frequency. Clearly label each curve.



Question 2-1 For what relative values of X_L and X_C will the total impedance of the circuit, *Z*, be a minimum? Explain your reasoning.

2. On the axes above, mark and label the frequency where Z is a minimum.

Question 2-2 At the frequency you labeled, will the value of the peak current, I_{max} , in the circuit be a maximum or minimum? What about the value of the peak voltage, V_{max} , across the resistor? Explain.

Note: The point you identified in step 2 is the *resonant frequency*. Label it with the symbol f_0 . The resonant frequency is the frequency at which the impedance of the series combination of a resistor, capacitor and inductor is minimal. This occurs at a frequency where the values of X_L and X_C are equal.

- 3. On the axes above (after step 1) draw a curve that qualitatively represents $X_{\rm L}$ $X_{\rm C}$ vs. frequency. Be sure to label it.
- 4. Use your results from above to determine the general mathematical expression for the resonant frequency, f_0 , as a function of *L* and *C*. (**Hint**: you will need the expressions for X_C and X_L given to you in step 1)

Equation for f_0 :

You will now test your predictions.

Activity 2-1: The Resonant Frequency of a Series RLC Circuit.

- 1. Open the experiment file **RLC Resonance**.
- 2. Adjust the scope display to 1 V/div and 1 ms/div.
- 3. Connect the circuit with resistor, capacitor, inductor, signal generator and probe shown in Figure 2-1.
- 4. Set the signal generator to a frequency of 200 Hz and amplitude of 5 V.
- 5. Press **Start** to begin taking data
- 6. Use the **Smart Tool** to determine the peak voltage, V_{max} .
- 7. Enter the data in the first row of **Table 2-1**.

$f_{signal} \left(\mathrm{Hz} \right)$	$V_{max}(\mathbf{V})$
200	
700	
1,200	
1,700	
2,200	
2,700	
3,200	

- 8. Repeat steps 5 through 7 for the other frequencies in **Table 2-1**. Be sure that the amplitude of the signal generator is always 5 V.
- 9. Calculate the resonant frequency for your circuit. Show your calculations. Use the formula from step 4 and the actual values of the capacitance and inductance.)

 $f_{\text{resonance}} =$ ____Hz CALCULATED

10. Measure the resonant frequency of the circuit to within a few Hz. To do this, press **Start** to begin taking data and slowly adjust the frequency of the signal generator until the peak voltage across the resistor is maximal. It may be helpful to use the scope display for this. (Use the results from Table 2-1 to help you locate the resonant frequency.)

 $f_{\text{resonance}} =$ _____Hz EXPERIMENTAL (Amplitude)

Question 2-3: How does this experimental value for the resonant frequency compare with your calculated one?

Activity 2-2: Phase in an RLC Circuit

In previous labs (and in this one), you investigated the phase relationship between the current and voltage in an AC circuit composed of a signal generator connected to one of the following circuit elements: a resistor, capacitor, or an inductor. You found that the current and voltage are in phase when the element connected to the signal generator is a resistor, the current leads the voltage with a capacitor, and the current lags the voltage with an inductor.

You also discovered that the reactances of capacitors and inductors change in predictable ways as the frequency of the signal changes, while the resistance of a resistor is constant – independent of the signal frequency. When considering relatively high or low signal frequencies in a simple RLC circuit, the circuit element (either capacitor or inductor) with the highest reactance is said to "dominate" because this element determines whether the current lags or leads the voltage. At resonance, the reactances of capacitor and inductor cancel, and do not contribute to the impedance of the circuit. The resistor then is said to dominate the circuit.

In this activity, you will explore the phase relationship between the applied voltage (signal generator voltage) and current in an RLC circuit.

Consider the RLC circuit shown below.

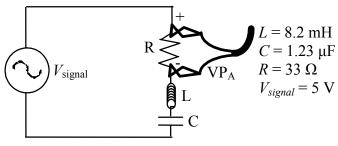


Figure 2-2: RLC series circuit

Question 2-4: Which circuit element (the resistor, inductor, or capacitor) dominates the circuit in Figure 2-2 at frequencies well below the resonant frequency? Explain.

Question 2-5: Which circuit element (the resistor, inductor, or capacitor) dominates the circuit in Figure 2-2 at frequencies well above the resonant frequency? Explain.

Question 2-6a: In the circuit in Figure 2-2, will the current through the *resistor* always be in phase with the voltage across the *resistor*, regardless of the frequency? Explain your reasoning.

Question 2-6b: If your answer to Question 2-6a was *no*, then which will lead for frequencies below the resonant frequency (current or voltage)? Which will lead for frequencies above the resonant frequency (current or voltage)?

Question 2-7a: In the circuit in Figure 2-2, will the current through the *resistor* always be in phase with applied voltage from the *signal generator*? Why or why not?

Question 2-7b: If your answer to Question 2-7a was *no*, then which will lead for frequencies below the resonant frequency (current or voltage)? Which will lead for frequencies above the resonant frequency (current or voltage)?

Test your predictions.

- 1. Open the experiment file called **RLC Phase**.
- 2. Connect the circuit shown in Figure 2-2.
- 3. Set the signal generator to a frequency 200 Hz below the resonant frequency you measured in Activity 2-1, and set the amplitude of the signal to 5 V.
- 4. Press Start to begin taking data.
- 5. Determine whether the current or applied voltage leads.

Question 2-8: Which leads – applied voltage, current or neither – when the AC signal frequency is lower than the resonant frequency? Were your predictions correct? Why or why not? Explain.

- 6. Set the signal generator to a frequency 200 Hz above the resonant frequency with the amplitude of signal still 5 V.
- 7. Determine whether the current or applied voltage leads.

Question 2-9: Which leads –applied voltage, current or neither – when the AC signal frequency is higher than the resonant frequency? Were your predictions correct? Why or why not? Explain.

Prediction 2-5: Which will lead for an applied signal at the resonant frequency (current or voltage or neither)?

- 8. Set the signal generator to the resonant frequency you measured in Activity 2-1, and set the amplitude of the signal to 5 V.
- 9. Determine whether the current or applied voltage leads.

Question 2-10: At resonance, does the current or applied voltage lead (or neither)?

10. Use this result to find the resonant frequency.

 $f_{\text{resonance}} =$ ____Hz EXPERIMENTAL (Phase)

Question 2-11: How does this experimental value for the resonant frequency with your calculated one?

Question 2-12: How does this experimental value for the resonant frequency compare with the one you determined by looking at the amplitude? Comment on the relative "sensitivities" of the two techniques.